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OPTIMAL STOP-LOSS REINSURANCE: A DEPENDENCE ANALYSIS

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Optimal stop-loss reinsurance: a dependence analysis

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Abstract

The stop-loss reinsurance is one of the most important reinsurance contracts in the insurance market. From the insurer point of view, it presents an interesting property: it is optimal if the criterion of minimizing the variance of the cost of the insurer is used. The aim of the paper is to contribute to the analysis of the stop-loss contract in one period from the point of view of the insurer and the reinsurer. Firstly, the influence of the parameters of the reinsurance contract on the correlation coefficient between the cost of the insurer and the cost of the reinsurer is studied. Secondly, the optimal stop-loss contract is obtained if the criterion used is the maximization of the joint survival probability of the insurer and the reinsurer in one period.

Keywords: Stop-loss premium, survival probabilities, reinsurance

1. Introduction

An insurance company may decide to sign a reinsurance contract either to assume greater risks or to protect the company. This reinsurance contract transfers part of the risks assumed by the insurer to the reinsurer in exchange of giving also a part of the premiums received from policyholders. Yet, reinsurance is the most important decision that an insurance company has to consider in order to reduce its underwriting risk. Two large groups of reinsurance contracts can be distinguished: the proportional and the non-proportional reinsurance. The proportional reinsurance includes two kinds of reinsurances known as quota-share and surplus. In the former, all the risks are transferred in the same proportion, while in the latter the proportion may vary. As regards the non-proportional reinsurances, the stop-loss and excess-loss contracts stand out. In both cases, the reinsurance offers protection when the aggregate claims exceed a certain agreed level.

The stop-loss reinsurance has been widely studied in the actuarial literature. If the criterion of minimizing the variance of the cost of the insurer is used, the stop-loss is the optimal reinsurance contract (Borch (1969)). From the point of view of the insurer, there are many studies in which a reinsurance contract is applied (see Centeno and Simões (2009) and the references therein). Indeed, during the last years, the joint analysis of the insurer and the reinsurer has gained significant attention (e.g. Dimitrova and Kaishev (2010), Castañer *et al.* (2013), Cai *et al.* (2013) and Salcedo-Sanz *et al.* (2014)).

The objective of this work is to contribute to the analysis of the stop-loss reinsurance in one period, from the joint point of view of the insurer and the reinsurer, in two aspects. The first one, consists on the calculus of the correlation coefficient between the costs of the insurer and the reinsurer in general and taking into account the different approximations to the distribution of

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the total cost. The second one, consists on the determination of the optimal stop-loss contract if the criterion is the maximization of the joint survival probability of the insurer and the reinsurer in one period.

The paper is organized as follows. Section 2 analyzes the expression of the correlation coefficient and the specific expressions for different distributions of the total cost, considering a stop-loss reinsurance with priority d. In Section 3, a maximum m in the stop-loss reinsurance is considered and the general and the specific expressions for the correlation coefficient are obtained. In Section 4, we introduce the probability of joint survival as a measure for the solvency for a reinsurance contract with priority d and reinsurance with d and m. In Section 5, the problem of finding the optimal reinsurance stop-loss if the criterion is the maximization of the joint survival probability is solved. In addition, a number of examples are presented. Section 6 closes the paper offering some final conclusions and remarks.

2. Correlation between the cost of the insurer and the cost of the reinsurer

In the stop-loss reinsurance contract with priority $d > 0$ the random variable (r.v.) total cost of claims in one period, S, is split between the cost of the insurer, SI, and the cost of the reinsurer, SR, with $S = SI + SR$, $SR = \max\{S - d, 0\}$ and $SI = min\{S, d\}$. The distribution functions of these two r.v., $F_{SI}(s) = P[SI \leq s]$ and $F_{SR}(s) = P[SR \leq s]$, can be calculated from the distribution function of S, $F_S(s) = P[S \leq s]$,

$$F_{SI}(s) = \begin{cases} F_S(s) & \text{if } \ s < d, \\ 1 & \text{if } \ s \geq d, \end{cases} \tag{2.1}$$

$$F_{SR}(s) = F_S(s + d). \tag{2.2}$$

The reinsurer can calculate the reinsurance premium with several premium principles. Most of these principles are based on the expectation of the total cost assumed by the reinsurer (Dickson (2005)). For instance, the net premium principle establishes that the premium is equal to the expectation of the cost. In the actuarial literature, the premium of an stop-loss contract calculated with the net premium principle is called the stop-loss premium. Let us define $\pi(d) = E[SR]$ as the stop-loss premium in a reinsurance stop-loss contract with priority d.

The r.v. cost of the reinsurer SR has the following two ordinary moments[1]:

$$\alpha_1(SR) = E[SR] = \int_d^{\infty} (s - d) f_S(s) ds = \int_d^{\infty} (1 - F_S(s)) ds, \tag{2.3}$$
$$\alpha_2(SR) = \int_d^{\infty} (s - d)^2 f_S(s) ds = 2 \int_d^{\infty} (s - d)(1 - F_S(s)) ds.$$

Hence, the variance is

$$V[SR] = \alpha_2(SR) - \alpha_1^2(SR) = E[SR](-2d - E[SR]) + 2 \int_d^{\infty} s(1 - F_S(s)) ds. \tag{2.4}$$

The expectation and the variance of the insurer cost SI can be calculated from those of S and SR, so:

$$\alpha_1(SI) = E[SI] = E\left[min(S, d)\right] = E\left[S\right] - E[SR],$$

[1]In order to obtain the expressions for the first two moments of the cost of the reinsurer it is necessary to take into account that $-f_S(s)ds = d(1 - F_S(s))$ and then apply integration by parts.

$$V[SI] = V[S] - V[SR] - 2Cov[SI, SR].$$

being

$$\begin{aligned} Cov[SI, SR] &= \int_d^\infty d(s-d)f_S(s)ds - E[SR](E[S] - E[SR]) \\ &= E[SR](d - E[S] + E[SR]). \end{aligned} \tag{2.5}$$

The correlation coefficient between SI and SR is

$$r(SI, SR) = \frac{Cov[SI, SR]}{\sqrt{V[SR](V[S] - V[SR] - 2Cov[SI, SR])}}. \tag{2.6}$$

In addition to the marginal analysis of the cost of the insurer and the reinsurer, we are interested in the bivariate r.v. (SI, SR). In a stop-loss reinsurance contract with priority d, the joint distribution function of the costs of the insurer and the reinsurer in one period is

$$P[SI \leq x, SR \leq y] = \begin{cases} P[S \leq x] & \text{if } x < d, \\ P[S \leq y + d] & \text{if } x \geq d > 0. \end{cases} \tag{2.7}$$

This r.v (SI, SR) is comonotone (Dhaene *et al.* (2002)) because SI and SR are increasing functions of the risk S. Then, there is a perfect positive dependence between the two marginal r.v. SI and SR and it is granted that the two parts that participate in the exchange of risk (the insurer and the reinsurer) increase their cost when the underlying risk increases. Hence, the correlation coefficient between SI and SR is the maximal one that can be attained between two random variables with the same marginal distributions, but it is not equal to one (this would be the case if one variable could be calculated as a linear function of the other, e.g. in proportional reinsurance)(Denuit and Charpentier (2004)). So, for a fixed d, $r(SI, SR)$ is the maximal one, but it is less than one in absolute value.

We are interested in the influence that the priority d has on the correlation coefficient. This influence depends on the distribution of the total cost in the period, S. Formulas (2.3) to (2.6) permit us to calculate the correlation coefficient.

The gamma distribution deserves special attention. It has been used in its version of two or three parameters to approximate the distribution of the total cost in a period as an alternative to the exact calculation through convolutions and to other approximations. In several papers (Bohman and Esscher (1963), Seal (1977), Gendron and Crepeau (1989)), the accuracy of the translated gamma approximation and the rest of approximations has been quantified. In this sense, Kaas (1993) uses the translated gamma approximation for the calculation of the stop-loss premium. In order to be self contained and to clarify the formulas that we use, we include in Section 2.1 a summary of the (translated) gamma distribution. Next, we indicate the explicit expressions of $\pi(d)$, $Cov[SI, SR]$ and $V[SR]$, which allow us calculating the coefficient of correlation for three different distributions or approximations for the total cost in a period: gamma with two parameters, translated gamma and Normal. As it is a simple calculation, we do not include the processes for obtaining these expressions.

2.1. Statistical summary

The gamma distribution with three parameters (or Pearson Type III) is also known as the translated gamma distribution, with one of its parameters interpreted as follows. If $X \sim Ga(\alpha, \beta, \gamma)$, with $\alpha > 0$, $\beta > 0$ and $\gamma \in \Re$, its density function is

$$f_X(x) = \frac{(x-\gamma)^{\alpha-1} e^{\frac{-(x-\gamma)}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, x > \gamma, \tag{2.8}$$

being γ, precisely, the parameter of translation. If $\gamma = 0$, the gamma distribution with two parameters is obtained, $X \sim Ga(\alpha, \beta)$ with $\alpha > 0$ and $\beta > 0$. The standard form of the distribution is obtained if, in addition, $\beta = 1$. Then, $X \sim Ga(\alpha)$, with $\alpha > 0$.

The gamma distribution with three parameters can be calculated through a gamma distribution with two or with one parameter (the standard form). Let $X \sim Ga(\alpha, \beta, \gamma)$, if $Y = (X-\gamma)/\beta$, then, $Y \sim Ga(\alpha)$, and also, $X = Y\beta + \gamma$. If $Z = X - \gamma$, then, $Z \sim Ga(\alpha, \beta)$, and the next relations are met,

$$X = Z + \gamma, Y = \frac{Z}{\beta}.$$

Recall that the moments and measures of X, Y and Z, are related as shown in Table 1

Table 1: Some characteristics of the gamma distribution

	$Y \sim Ga(\alpha)$	$Z \sim Ga(\alpha, \beta)$	$X \sim Ga(\alpha, \beta, \gamma)$
Mean μ_1	α	$\alpha\beta$	$\alpha\beta + \gamma$
Variance μ_2	α	$\alpha\beta^2$	$\alpha\beta^2$
μ_3	2α	$2\alpha\beta^3$	$2\alpha\beta^3$
Skewness γ_1	$\frac{2}{\sqrt{\alpha}}$	$\frac{2}{\sqrt{\alpha}}$	$\frac{2}{\sqrt{\alpha}}$

The parameters of $X \sim Ga(\alpha, \beta, \gamma)$, can be estimated by the moments' method:

$$\widehat{\alpha} = \frac{4}{\gamma_1^2(X)}, \widehat{\beta} = \frac{\mu_3(X)}{2\mu_2(X)}, \widehat{\gamma} = E[X] - \widehat{\alpha}\widehat{\beta}. \tag{2.9}$$

Taking into account Table 1, a variable $X \sim Ga(\alpha, \beta, \gamma)$, also meets the next relationship with the variable $Y \sim Ga(\alpha)$ (if the parameter α is estimated through the asymmetry of X, as in (2.9)),

$$X = \mu_1(X) + \mu_2^{0.5}(X)\frac{Y - \alpha}{\sqrt{\alpha}}.$$

Then,

$$\begin{aligned} P[X \leq x] &= P\left[\mu_1(X) + \mu_2^{0.5}(X)\frac{Y-\alpha}{\sqrt{\alpha}} \leq x\right] = P\left[Y \leq \alpha + \sqrt{\alpha}\frac{x - \mu_1(X)}{\mu_2^{0.5}(X)}\right] \\ &= Ga\left(\alpha + \sqrt{\alpha}\frac{x-\mu_1(X)}{\mu_2^{0.5}(X)}; \alpha\right), \end{aligned} \tag{2.10}$$

being $Ga(y; \alpha) = P[Y \leq y]$ with $Y \sim Ga(\alpha)$. Or alternatively,

$$P[X \leq x] = P[Z + \gamma \leq x] = P[Z \leq x - \gamma] = Ga(x - \gamma; \alpha, \beta), \tag{2.11}$$

being $Ga(z; \alpha, \beta) = P[Z \leq z]$ with $Z \sim Ga(\alpha, \beta)$.

2.2. Gamma distribution (with two parameters)

Assume $S \sim Ga(\alpha, \beta)$, with $\alpha > 0$ and $\beta > 0$. The density function and the distribution function are, respectively,

$$\begin{aligned} f_S(s) &= \frac{s^{\alpha-1}e^{-\frac{s}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, \quad s > 0, \\ F_S(s) &= Ga(s; \alpha, \beta), \quad s > 0. \end{aligned}$$

Hence, in this case we have

$$\pi(d) = \alpha\beta\left(1 - Ga(d;\alpha+1,\beta)\right) - d\left(1 - Ga(d;\alpha,\beta)\right),$$

$$\begin{aligned} Cov\left[SI, SR\right] &= \left[\alpha\beta\left(1 - Ga(d;\alpha+1,\beta)\right) - d\left(1 - Ga(d;\alpha,\beta)\right)\right] \\ &\times \left[-\alpha\beta Ga(d;\alpha+1,\beta) + d\ Ga(d;\alpha,\beta)\right] \end{aligned}$$

and

$$\begin{aligned} V\left[SR\right] &= \pi(d)\left(-2d - \pi(d)\right) - d^2\left(1 - Ga(d;\alpha,\beta)\right) \\ &+ (\alpha+1)\alpha\beta^2\left(1 - Ga(d;\alpha+2,\beta)\right). \end{aligned}$$

2.3. Translated gamma distribution

Assume $S \sim Ga(\alpha,\beta,\gamma)$, with $\alpha > 0$, $\beta > 0$ and $\gamma \in \Re$. The density function and the distribution function are, respectively,

$$\begin{aligned} f_S(s) &= \frac{(s-\gamma)^{\alpha-1} e^{-\frac{s-\gamma}{\beta}}}{\beta^\alpha \Gamma(\alpha)}, \quad s > \gamma, \\ F_S(s) &= Ga(s;\alpha,\beta,\gamma), \quad s > \gamma. \end{aligned}$$

For the translated gamma approximation for the distribution of the total cost, we obtain two equivalent expressions for the stop-loss premium depending on the formula used, (2.10) or (2.11). First, from (2.10) we have,

$$\pi(d) = E\left[(S-d)_+\right] \approx \frac{\mu_2^{0.5}(S)}{\sqrt{\alpha}}\left[d' f(d';\alpha) + (\alpha - d')(1 - Ga(d';\alpha))\right], \tag{2.12}$$

being $d' = \alpha + \sqrt{\alpha}\left(\frac{d-\mu_1(S)}{\mu_2^{0.5}(S)}\right)$ and $f(d';\alpha)$, the density function of $Y \sim Ga(\alpha)$ in d'.

Second, from (2.11) we have,

$$\begin{aligned} \pi(d) &= E\left[(S-d)_+\right] \approx \alpha\beta\left(1 - Ga(d-\gamma;\alpha+1,\beta)\right) \\ &- (d-\gamma)(1 - Ga(d-\gamma;\alpha,\beta)), \end{aligned} \tag{2.13}$$

Expression (2.12) can be found in Kaas (1993) as a particular case of the ordinary moments of the cost of the reinsurer.

From (2.4), (2.5) and (2.13) the $Cov[SI, SR]$ can be easily calculated, and the expression of the variance of SR is

$$\begin{aligned} V[SR] &= \pi(d)\left(-2d - \pi(d)\right) + 2\alpha\beta\gamma\left(1 - Ga(d-\gamma;\alpha+1,\beta)\right) \\ &+ (\alpha+1)\alpha\beta\left(1 - Ga(d-\gamma;\alpha+2,\beta)\right) + \left(\gamma^2 - d^2\right)\left(1 - Ga(d-\gamma;\alpha,\beta)\right). \end{aligned}$$

2.4. Normal distribution

Assume $S \sim N(\mu,\sigma)$, with $\mu = E\left[S\right]$ and $\sigma^2 = V\left[S\right] > 0$. The density and distribution functions are, respectively, in terms of the distribution of $N(0,1)$,

$$\begin{aligned} f_S(s) &= \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(s-\mu)^2}{2\sigma^2}}, \\ F_S(s) &= \Phi\left(\frac{s-\mu}{\sigma}\right), \end{aligned}$$

and then,

$$\pi(d) = \sigma\phi\left(\frac{d-\mu}{\sigma}\right) + (\mu - d)\left(1 - \Phi\left(\frac{d-\mu}{\sigma}\right)\right),$$

$$\begin{aligned} Cov\left[SI, SR\right] &= \left[\sigma\phi\left(\frac{d-\mu}{\sigma}\right) + (\mu - d)\left(1 - \Phi\left(\frac{d-\mu}{\sigma}\right)\right)\right] \\ &\times \left[\sigma\phi\left(\frac{d-\mu}{\sigma}\right) - (\mu - d)\left(\Phi\left(\frac{d-\mu}{\sigma}\right)\right)\right] \end{aligned}$$

and

$$V\left[SR\right] = -\sigma(d-\mu)\phi\left(\frac{d-\mu}{\sigma}\right) - \pi(d)^2 + \left((\mu-d)^2 + \sigma^2\right)\left(1 - \Phi\left(\frac{d-\mu}{\sigma}\right)\right).$$

Example 1. *We assume that the total cost of a period has the following characteristics:* $E\left[S\right] = 1$*,* $V\left[S\right] = 2$ *and skewness* $\gamma_1(S) = \frac{3}{\sqrt{2}}$*. In Table 2 we show the mean and the variance of the costs for the insurer and the reinsurer as well as the coefficient of correlation as a function of the parameter* d *of the stop-loss reinsurance using the translated gamma approximation.*

Table 2: Mean and variance of the costs for the insurer and the reinsurer and the coefficient of correlation in a stop-loss reinsurance with priority d (Translated gamma approximation)

d	$E\left[SR\right]$	$V\left[SR\right]$	$E\left[SI\right]$	$V\left[SI\right]$	$r(SI, SR)$
0.1	0.9728	1.8413	0.0272	0.0170	0.4001
0.2	0.9057	1.7796	0.0943	0.0289	0.4221
0.3	0.8435	1.7136	0.1565	0.0444	0.4390
0.4	0.7857	1.6448	0.2143	0.0633	0.4523
0.5	0.7321	1.5745	0.2679	0.0857	0.4627
0.6	0.6822	1.5037	0.3178	0.1112	0.4708
0.7	0.6359	1.4330	0.3641	0.1398	0.4771
0.8	0.5928	1.3632	0.4072	0.1712	0.4819
0.9	0.5526	1.2946	0.4474	0.2051	0.4855
1.0	0.5153	1.2278	0.4847	0.2412	0.4880
1.1	0.4805	1.1628	0.5195	0.2792	0.4896
1.2	0.4482	1.1001	0.5518	0.3189	0.4904
1.3	0.4180	1.0396	0.5820	0.3601	0.4906
1.4	0.3899	0.9815	0.6101	0.4024	0.4901
1.5	0.3638	0.9259	0.6362	0.4457	0.4892
1.6	0.3394	0.8728	0.6606	0.4896	0.4877
1.7	0.3167	0.8221	0.6833	0.5340	0.4859
1.8	0.2955	0.7739	0.7045	0.5787	0.4837
1.9	0.2757	0.7280	0.7243	0.6236	0.4812
2.0	0.2573	0.6846	0.7427	0.6683	0.4783
2.1	0.2402	0.6434	0.7598	0.7129	0.4752
2.2	0.2241	0.6044	0.7759	0.7572	0.4719
2.3	0.2092	0.5676	0.7908	0.8009	0.4683
2.4	0.1953	0.5328	0.8047	0.8442	0.4645
2.5	0.1823	0.4999	0.8177	0.8867	0.4606
2.6	0.1702	0.4690	0.8298	0.9285	0.4565
2.7	0.1589	0.4398	0.8411	0.9696	0.4522
2.8	0.1483	0.4123	0.8517	1.0097	0.4479
2.9	0.1385	0.3865	0.8615	1.0489	0.4433
3.0	0.1293	0.3622	0.8707	1.0872	0.4387
4.0	0.0652	0.1874	0.9348	1.4128	0.3884
5.0	0.0330	0.0961	0.9670	1.6380	0.3352
10.0	0.0011	0.0033	0.9989	1.9766	0.1241

In Table 3, we show the evolution of the coefficient of correlation between the costs of the insurer and the reinsurer as a function of the priority of the reinsurance for the gamma, translated gamma and normal approximations. In reference to the coefficient of correlation, we find that, for the three approximations, it follows the same trends: first it increases and then decreases, reaching a maximum for some priority 2.1, 1.3 *and* 1 *for the gamma, the translated gamma and the normal approximation, respectively.*

Table 3: Coefficient of correlation in a stop-loss reinsurance with priority d

d	Gamma approximation	Translated gamma approximation	Normal approximation
0.1	0.0298	0.4001	0.4385
0.2	0.0621	0.4221	0.4444
0.3	0.0957	0.4390	0.4496
0.4	0.1303	0.4523	0.4542
0.5	0.1651	0.4627	0.4581
0.6	0.1999	0.4708	0.4613
0.7	0.2341	0.4771	0.4637
0.8	0.2674	0.4819	0.4655
0.9	0.2993	0.4855	0.4666
1.0	0.3295	0.4880	**0.4669**
1.1	0.3577	0.4896	0.4666
1.2	0.3835	0.4904	0.4655
1.3	0.4067	**0.4906**	0.4637
1.4	0.4272	0.4901	0.4613
1.5	0.4449	0.4892	0.4581
1.6	0.4597	0.4877	0.4542
1.7	0.4717	0.4859	0.4496
1.8	0.4808	0.4837	0.4444
1.9	0.4873	0.4812	0.4385
2.0	0.4913	0.4783	0.4319
2.1	**0.4929**	0.4752	0.4247
2.2	0.4924	0.4719	0.4169
2.3	0.4900	0.4683	0.4085
2.4	0.4859	0.4645	0.3995
2.5	0.4803	0.4606	0.3900
2.6	0.4734	0.4565	0.3800
2.7	0.4654	0.4522	0.3694
2.8	0.4565	0.4479	0.3585
2.9	0.4470	0.4433	0.3471
3.0	0.4368	0.4387	0.3353
4.0	0.3277	0.3884	0.2089
5.0	0.2368	0.3352	0.1003
10.0	0.0528	0.1241	0.0000451

3. Correlation between the cost of the insurer and the cost of the reinsurer in a stop-loss reinsurance with maximum

The stop-loss reinsurance contract can include a priority d and a maximum m, $m > d > 0$. In this case,

$$SR(d,m) = \min\{m-d, \max\{S-d,0\}\},$$

$$SI(d,m) = \min\{S,d\} + \max\{S-m,0\}.$$

The distribution functions of these two r.v. are

$$F_{SI(d,m)}(s) = \begin{cases} F_S(s) & \text{if } s < d, \\ F_S(s+m-d) & \text{if } s \geq d \end{cases} \tag{3.1}$$

and

$$F_{SR(d,m)}(s) = \begin{cases} F_S(s+d) & \text{if } s < m-d, \\ 1 & \text{if } s \geq m-d. \end{cases} \tag{3.2}$$

Let $\pi(d,m) = E[SR(d,m)]$ be the stop-loss premium, that is the reinsurance premium calculated with the net premium principle. It can be calculated from the premiums of a stop-loss reinsurance with priorities d and m, $\pi(d,m) = \pi(d) - \pi(m)$.

The second ordinary moment $\alpha_2(SR(d,m))$, is

$$\begin{aligned} \alpha_2(SR(d,m)) &= \int_d^m (s-d)^2 f_S(s)ds + \int_m^\infty (m-d)^2 f_S(s)ds \\ &= \int_d^\infty (s-d)^2 f_S(s)ds - \int_m^\infty (s-d)^2 f_S(s)ds + \int_m^\infty (m-d)^2 f_S(s)ds \\ &= \alpha_2(SR(d)) - \int_m^\infty ((s-d)^2 - (m-d)^2) f_S(s)ds \\ &= \alpha_2(SR(d)) - \alpha_2(SR(m)) - 2(m-d)\pi(m), \end{aligned}$$

where the last equality follows taking into account that $(s-d)^2 - (m-d)^2 = (s-m)^2 + 2(s-m)(m-d)$.

Hence, the variance $V[SR(d,m)]$, is:

$$\begin{aligned} V[SR(d,m)] &= \alpha_2(SR(d,m)) - \alpha_1(SR(d,m))^2 \\ &= \alpha_2(SR(d)) - \alpha_2(SR(m)) - 2(m-d)\pi(m) - (\pi(d) - \pi(m))^2 \\ &= V[SR(d)] - V[SR(m)] + 2\pi(m)(\pi(d) + d - \pi(m) - m). \end{aligned}$$

The covariance between the costs of the insurer and the reinsurer is:

$$\begin{aligned} Cov[SI(d,m), SR(d,m)] &= \int_d^m d(s-d) f_S(s)ds + \int_m^\infty (m-d)(s-m+d) f_S(s)ds \\ &= \int_d^\infty d(s-d) f_S(s)ds \\ &- \int_m^\infty (d(s-d) - (m-d)(s-m+d)) f_S(s)ds \\ &= Cov[SI(d), SR(d)] - \int_m^\infty ((s-m)(2d-m)) f_S(s)ds \\ &= Cov[SI(d), SR(d)] - (2d-m)\pi(m). \end{aligned}$$

where the last but one equality follows taking into account that $d(s-d) - (m-d)(s-m+d) = (s-m)(2d-m)$.

So, in order to calculate the expectation and the variance of the costs of the insurer and the reinsurer, and the covariance if the stop-loss has a maximum, we only need the expressions of a stop-loss without maximum, which have been obtained in Section 2.

The distribution function of the bivariate r.v. $(SI(d,m), SR(d,m))$ is

$$P\left[SI(d,m) \le x, SR(d,m) \le y\right] = \begin{cases} P\left[S \le x\right] & \text{if } \ x < d, \\ P\left[S \le d\right] & \text{if } \ x \ge d \text{ and } \ y = 0, \\ P\left[S \le y + d\right] & \text{if } \ x \ge d \text{ and } \ 0 < y < m - d, \\ P\left[S \le m\right] & \text{if } \ x = d \text{ and } \ y \ge m - d, \\ P\left[S \le x + m - d\right] & \text{if } \ x > d \text{ and } \ y \ge m - d. \end{cases} \tag{3.3}$$

Example 2. *Using the data for total costs in Example 1, we calculate the variation of the mean and the variance of the cost of the insurer and the reinsurer with respect to the maximum* m *for three different values of the priority (*0.2*,* 0.8 *and* 1.3*). In turn, Figures 1 and 2 show the evolution of such magnitudes if the translated gamma approximation is used. Finally, in Figure 3, we present the coefficient of correlation as a function of the maximum for the three priorities previously indicated.*

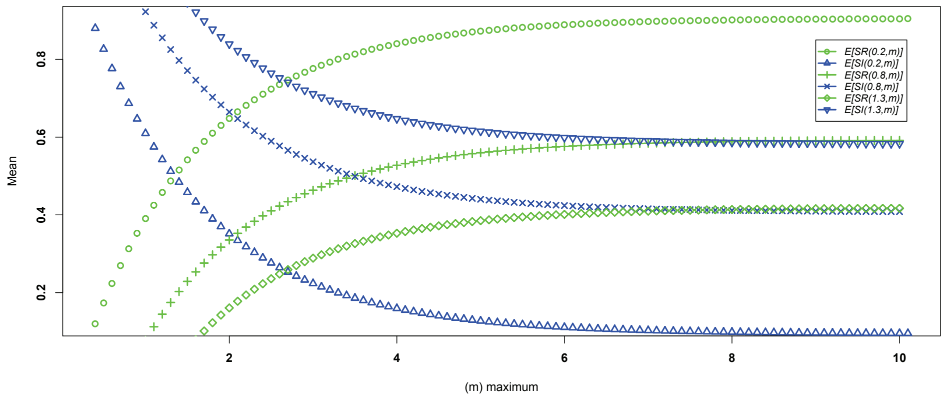

Figure 1: Mean of the cost of the insurer and the reinsurer as a function of the maximum, for different values of d (Translated gamma approximation)

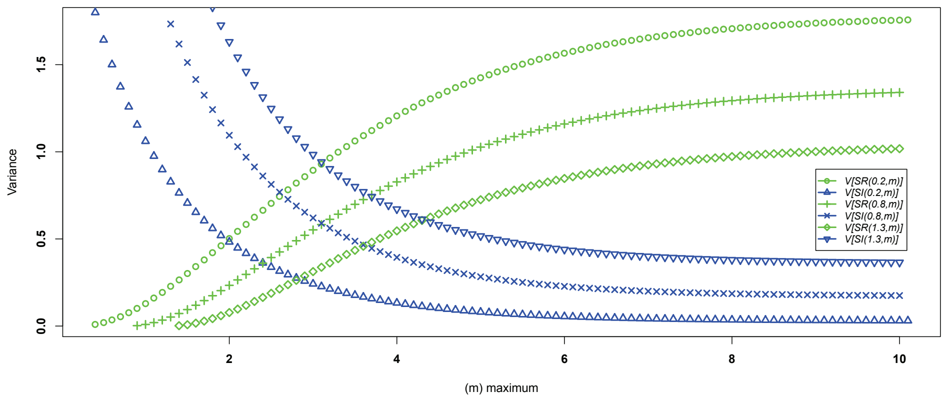

Figure 2: Variance of the cost of the insurer and the reinsurer as a function of the maximum, for different values of d (Translated gamma approximation)

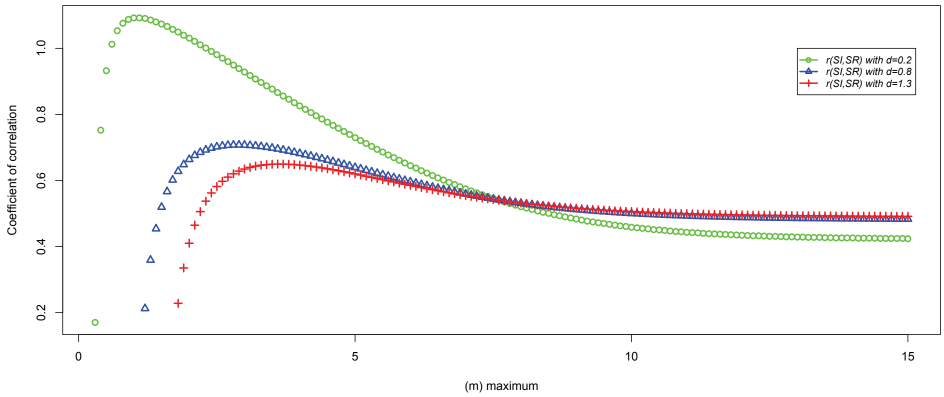

Figure 3: Coefficient of correlation between the costs of the insurer and the reinsurer for different options of stop-loss (Translated gamma approximation)

For some combinations of priority and maximum, the translated gamma approximation sheds values higher than one for the coefficient of correlation. Hence, if the objective is to calculate the coefficient of correlation, the translated gamma is not a good approximation for any value of the priority and the maximum.

4. Survival probabilities in one period

The survival probability is one of the most important measures of the solvency of an insurer/reinsurer. The survival probability in one period of an insurer considering only the underwriting risk, can be calculated knowing the distribution of the cost of the insurer, the reserves at the beginning of the period and the premium earned by the insurer to cover the insured risk. If a stop-loss reinsurance contract is agreed, the survival probability of the insurer is obviously different and needs to be calculated again with the new parameters; but, as in this case, if the payment of the claims depends on the two parts, the joint survival probability of insurer and reinsurer is also a quantity of interest.

Let $PT > 0$ be the premium earned by the insurer in the period; let $PR > 0$ be the reinsurer's premium; let $uI \geq 0$ and $uR \geq 0$ be the initial reserves of the insurer and the reinsurer, respectively. It is then possible to incorporate in the model an economic constraint: the reinsurer's premium must be less than the premium earned by the insurer in the period, $0 < PR < PT$.

4.1. Stop-loss reinsurance with priority d

The survival probability of the insurer, $\phi_I(uI, d, PR, PT)$, is

$$\phi_I(uI, d, PR, PT) = P\left[uI + PT - PR - SI \geq 0\right] = P\left[SI \leq uI + PT - PR\right] = F_{SI}(uI+PT-PR)$$

and from (2.1),

$$\phi_I(uI, d, PR, PT) = \begin{cases} F_S(uI + PT - PR) & \text{if } \; uI + PT - PR < d, \\ 1 & \text{if } \; uI + PT - PR \geq d. \end{cases} \tag{4.1}$$

The survival probability of the reinsurer, $\phi_R(uR, d, PR)$, is

$$\phi_R(uR, d, PR) = P\left[uR + PR - SR \geq 0\right] = P\left[SR \leq uR + PR\right] = F_{SR}(uR + PR)$$

and from (2.2),

$$\phi_R(uR, d, PR) = F_S(uR + PR + d). \tag{4.2}$$

The joint survival probability of the insurer and the reinsurer, $\phi_{I,R}(uI, uR, d, PR, PT)$, is

$$\phi_{I,R}(uI, uR, d, PR, PT) = P\left[SI \le uI + PT - PR, SR \le uR + PR\right]$$

and from (2.7),

$$\phi_{I,R}(uI, uR, d, PR, PT) = \begin{cases} F_S(uI + PT - PR) & \text{if } \ uI + PT - PR < d, \\ F_S(uR + PR + d) & \text{if } \ uI + PT - PR \ge d. \end{cases} \tag{4.3}$$

4.2. Stop-loss reinsurance with priority d and maximum m

The survival probability of the insurer, $\phi_I(uI, d, m, PR, PT)$, is

$$\phi_I(uI, d, m, PR, PT) = F_{SI(d,m)}(uI + PT - PR)$$

and from (3.1)

$$\phi_I(uI, d, m, PR, PT) = \begin{cases} F_S(uI + PT - PR) & \text{if } \ uI + PT - PR < d, \\ F_S(uI + PT - PR + m - d) & \text{if } \ uI + PT - PR \ge d. \end{cases}$$

The survival probability of the reinsurer, $\phi_R(uR, d, m, PR)$, is

$$\phi_R(uR, d, m, PR) = F_{SR(d,m)}(uR + PR)$$

and from (3.2)

$$\phi_R(uR, d, m, PR) = \begin{cases} F_S(uR + PR + d) & \text{if } \ uR + PR < m - d, \\ 1 & \text{if } \ uR + PR \ge m - d. \end{cases}$$

The joint survival probability of the insurer and the reinsurer, $\phi_{I,R}(uI, uR, d, m, PR, PT)$, is

$$\phi_{I,R}(uI, uR, d, m, PR, PT) = P\left[SI \le uI + PT - PR, SR \le uR + PR\right]$$

and from (3.3)

$$\phi_{I,R}(uI,uR,d,m,PR,PT) = \begin{cases} F_S(uI+PT-PR) & \text{if } \ uI+PT-PR<d, \\ F_S(uR+PR+d) & \text{if } \ uI+PT-PR\ge d \text{ and } \ uR+PR<m-d, \\ F_S(m) & \text{if } \ uI+PT-PR=d \text{ and } \ uR+PR\ge m-d, \\ F_S(uI+PT-PR+m-d) & \text{if } \ uI+PT-PR>d \text{ and } \ uR+PR\ge m-d. \end{cases} \tag{4.4}$$

5. Optimal joint survival probability in one period

In this section, we are interested in solving two different optimization problems related with the joint survival probability of the insurer and the reinsurer in one period.

In the first optimization problem, the reinsurance premium is fixed (as it is the total premium PT) and so are the initial values of the reserves of the insurer and the reinsurer. In addition, the parameters of the reinsurance maximize the probability of the joint survival probability. This probability is a function of the parameters of the reinsurance, d or d and m. Propositions 5.1 and 5.3 solve this problem.

It is usually considered that PR is a function of the parameters of the stop-loss reinsurance (d, m) and the total cost S. In that instance, the reinsurer would apply for the calculation of the premium some of the usual criteria, for instance, the expected value, variance and standard deviation principles (for more details see Kaas *et al.* (2008)). We adopt as a criterion for the calculation of the reinsurer's premiums the maximization of the joint survival probability, given as fixed both the values of the parameters of the reinsurance contract and the initial values of the reserves of the insurer and the reinsurer. Then, in the second optimization problem, the joint survival probability is considered to be a function of the reinsurance premium, PR. Propositions 5.2 and 5.4 tackle this problem.

Proposition 5.1. *In a stop-loss reinsurance with priority d, the program*

$$\max_{d} \phi_{I,R}(uI, uR, d, PR, PT) \text{ subject to } 0 < d$$

has as a maximum value $\phi^*_{I,R}(uI, uR, PR, PT) = F_S\,(uI + uR + PT)$, being the optimal point $d^*(uI, uR, PR, PT) = uI + PT - PR$.

Proof. The joint survival probability to be maximized, (4.3), is a step function built with the distribution function of the total cost. Since $F_S(x)$ is increasing in x and $uI + PT - PR < d < uR + PR + d$, for all $d > uI + PT - PR$, $F_S(uI + PT - PR) \leq F_S(uR + PR + uI + PT - PR) = F_S(uR + uI + PT)$, then it is immediate that $\phi^*_{I,R}(uI, uR, PR, PT)$ is attained at $d^*(uI, uR, PR, PT) = uI + PT - PR$. □

Remark 1 (Proposition 5.1). *In Figure 4, we plot the two-step function indicating the argument of the distribution function of the total cost in (4.3), as a function of d.*

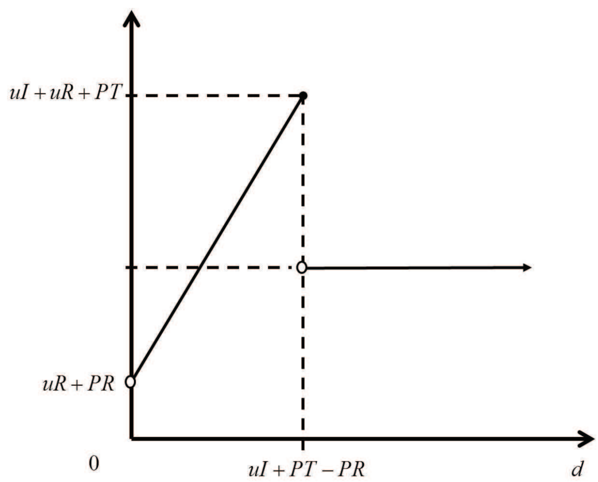

Figure 4: the argument of the distribution function of the total cost in (4.3) as a function of d

Remark 2 (Proposition 5.1). *For this optimal reinsurance, in which the maximum joint survival probability of the insurer and the reinsurer is obtained, the individual survival probability of the insurer (4.1) is $\phi_I(uI, uI{+}PT{-}PR, PR, PT) = 1$, whereas the individual survival probability of the reinsurer (4.2) is $\phi_R(uR, uI{+}PT{-}PR, PR) = F_S(uI{+}uR{+}PT) = \phi^*_{I,R}(uI, uR, PR, PT)$. Hence, the insurer, with this optimal reinsurance, increases his/her individual survival probability (compared to the absence of reinsurance) in* $(1 - P\,[S \leq uI + PT]) > 0$.

Remark 3 (Proposition 5.1). *If the initial capitals of the insurer and the reinsurer are zero, then the maximum joint survival probability is obtained when the priority d is equal to the net premium of the insurer.*

Proposition 5.2. *In a stop-loss reinsurance with priority d, the program*

$$\max_{PR} \phi_{I,R}(uI, uR, d, PR, PT) \text{ subject to } 0 < PR < PT$$

only provides a solution if $uI < d < uI+PT$, being in that case the maximum value $\phi^*_{I,R}(uI, uR, d, PT) = F_S\,(uI+uR+PT)$, which is reached for $PR^*(uI, uR, d, PT) = uI + PT - d$.

Proof. It is developed in a similar way as in Proposition 5.1. Since $F_S(x)$ is increasing in x, if $d \in (uI, uI+PT)$, for all $0 < PR \leq uI+PT-d$, $F_S(uR+uI+PT-d+d) = F_S(uR+uI+PT) \geq F_S(uR+PR+d)$ and for all $uI+PT-d < PR < PT$, $F_S(uI+uR+PT) > F_S(uI+PT-PR)$. If $d > uI + PT$, for all $0 < PR < PT$, $F_S(uI + PT - PR)$ does not have a maximum. If $d < uI$, for all $0 < PR < PT$, $F_S(uR + PR + d)$ does not have a maximum. Then, the program provides a solution only if $uI < d < uI + PT$ and $\phi^*_{I,R}(uI, uR, d, PT)$ is attained at $PR^*(uI, uR, d, PT) = uI + PT - d$. □

Remark 4 (Proposition 5.2). *In Figure 5, we plot the two-step function indicating the argument of the distribution function of the total in (4.3), as a function of PR when $uI < d < uI + PT$.*

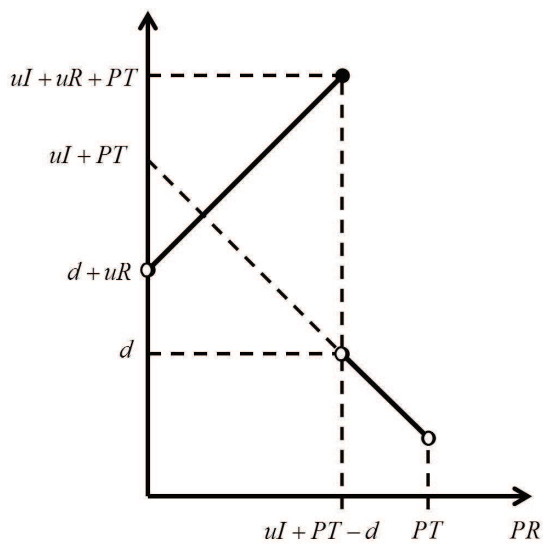

Figure 5: The argument of the distribution function of the total cost in (4.3) as a function of PR when $uI < d < uI + PT$

Proposition 5.3. *In a stop-loss reinsurance with priority d and maximum m, the program*

$$\max_{(d,m)} \phi_{I,R}(uI, uR, d, m, PR, PT) \text{ subject to } 0 < d < m$$

has a maximum value $\phi^*_{I,R}(uI, uR, PR, PT) = F_S\,(uI + uR + PT)$. This maximum is attained at the non-convex set

$$\left\{(d, m) \in \Re^2_+ \mid d \leq uI + PT - PR \text{ and } \; m = uR + PR + d\right\} \cup \left\{(d, m) \in \Re^2_+ \mid d = uI + PT - PR \text{ and } \; m > uR + PR + d\right\}$$

Proof. The joint survival probability to be maximized now is (4.4), a piecewise function built with the distribution function of the total cost. Since $F_S(x)$ is increasing in x, for all $(d,m) \in \Re^2_+$ such that $d \leq uI + PT - PR$ and $m > uR + PR + d$, $F_S(uR + PR + d) \leq F_S(uR + PR + uI + PT - PR) = F_S(uR + uI + PT)$. For all $(d,m) \in \Re^2_+$ such that $d < uI + PT - PR$ and $m \leq uR+PR+d$, $F_S(uI+PT-PR+m-d) \leq F_S(uI+PT-PR+uR+PR) = F_S(uI+uR+PT)$. Taking into account that $F_S(uI + uR + PT) > F_S(uI + PT - PR)$, the proof is completed. □

Remark 5 (Proposition 5.3). *In Figure 6, we plot the step function indicating the argument of the distribution function of the total cost in (4.4) as a function of d and m and its level curves. For $PT = 1$, $PR = 0.4$ and $uI = uR = 0$, the maximum value is 1 and the set of optimal points are $\{d \leq 0.6 \text{ and } \ m = 0.4 + d\} \cup \{d = 0.6 \text{ and } \ m > 0.4 + d\}$.*

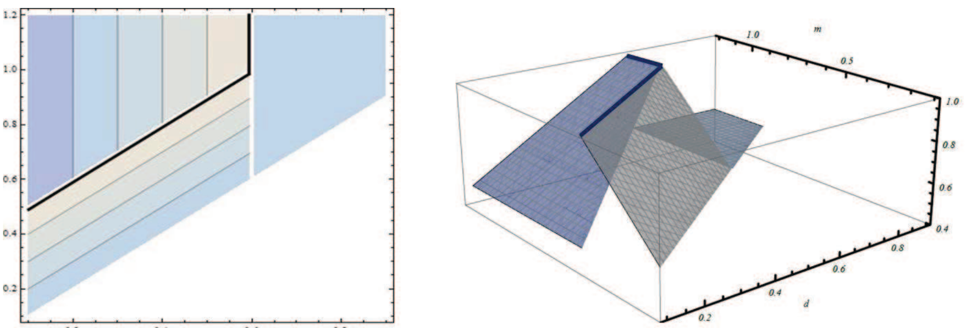

Figure 6: The argument of the distribution function of the total cost in (4.4) as a function of d and m (right graph) and its level curves (left graph) (for $PT = 1$, $PR = 0.4$ and $uI = uR = 0$)

Proposition 5.4. *In a stop-loss reinsurance with priority d and maximum m, the program*

$$\max_{PR} \phi_{I,R}(uI, uR, d, m, PR, PT) \text{ subject to } 0 < PR < PT$$

only provides solutions if

$$((uI<d<uI+PT)\cap(m\geq uI+uR+PT))\cup((m<uI+uR+PT)\cap(PT+uR>m-d>uR)).$$

In that case, the maximum value is $\phi^*_{I,R}(uI, uR, d, m, PT) = F_S\,(uI + uR + PT)$, being the optimal premiums of the reinsurer

$$PR^*(uI,uR,d,m,PT) = \begin{cases} uI+PT-d & \text{if } \ ((uI<d<uI+PT)\cap(m\geq uI+uR+PT)), \\ m-d-uR & \text{if } \ ((m<uI+uR+PT)\cap(PT+uR>m-d>uR)). \end{cases}$$

Proof. Taking into account (4.4) and that $0 < PR < PT$, lets first consider the case that $d \in (uI, uI+PT)$. If $uI + PT - d < m - d - uR$, for all $0 < PR \leq uI + PT - d$, $F_S(uR + uI + PT - d + d) = F_S(uR + uI + PT) \geq F_S(uR + PR + d)$ and for all $uI + PT - d < PR < PT$, $F_S(uI+uR+PT) > F_S(uI+PT-PR)$. If $uI+PT-d = m-d-uR$, for all $0 < PR \leq uI+PT-d$, $F_S(m) = F_S(uI + uR + PT) > F_S(uR + PR + d)$ and for all $uI + PT - d < PR < PT$, $F_S(uI + uR + PT) > F_S(uI + PT - PR)$.

Secondly, lets consider that $(m-d) \in (uR, uR+PT)$ and $uI+PT-d > m-d-uR$, for all $0 < PR \leq m-d-uR$, $F_S(uI+PT-m+d+uR+m-d) = F_S(uI+PT+uR) > F_S(uR+PR+d)$ and for all $PR > m-d-uR$, $F_S(uI+uR+PT) > F_S(uI+PT-PR+m-d) > F_S(uI+PT-PR)$.

It is then easy to demonstrate that for all the other possibles values of d and m, the maximum does not exist. □

Remark 6 (Proposition 5.4). *In Figure 7, the argument of the distribution function of the total cost in (4.4) is plotted as a function of PR for the values d and m for which the joint survival probability has a maximum. It can be divided into three cases depending on whether $uI + PT - d$ is less, equal or greater than $m - d - uR$.*

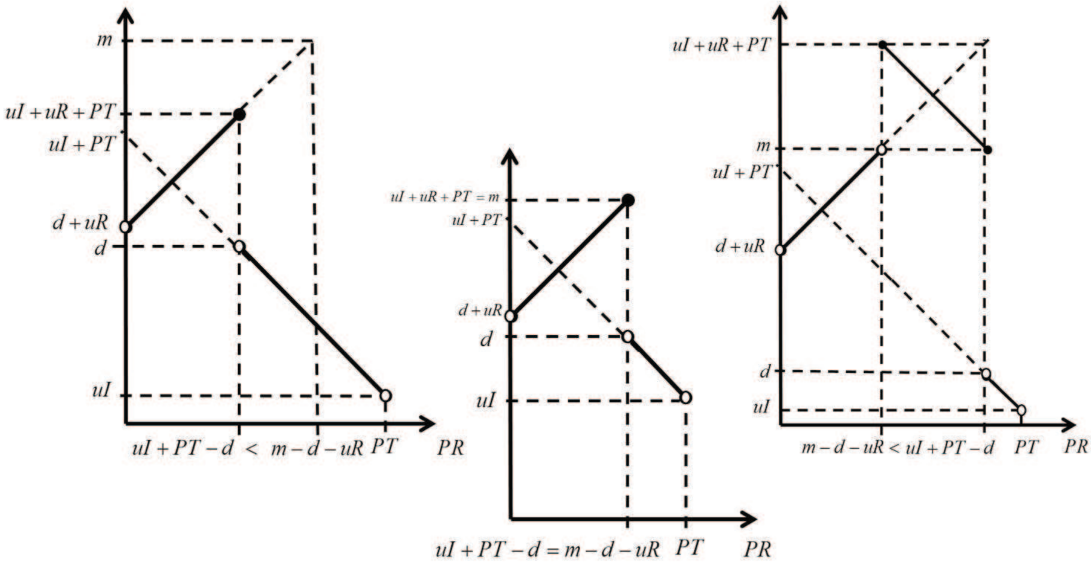

Figure 7: The argument of the distribution function of the total cost in (4.4) as a function of PR when $uI+PT-d \lesseqgtr m-d-uR$. The graph on the left considers $uI + PT - d < m - d - uR$; the graph on the middle considers $uI + PT - d = m - d - uR$ and the graph on the right considers $uI + PT - d > m - d - uR$.

From Propositions 5.1 to 5.4, the maximum joint survival probability (considering the constraints), when it exists, is equal to

$$F_S\left(uI + uR + PT\right).$$

From the first definition of ruin in a bivariate risk process (Castañer *et al.* (2013)), the joint survival probability equals to the minimum between the survival probability of the insurer and the survival probability of the reinsurer, and this is also true at the optimal points. Then, at the optimal points, the survival probability of the insurer or the reinsurer must be equal to $F_S\left(uI + uR + PT\right)$, and the other must be greater than this value. Table 4 includes the values of the survival probability of the insurer and the reinsurer at the points that maximize the joint survival probability.

Table 4: ϕ_I and ϕ_R at the optimal points for the different optimization problems

	Φ_I	Φ_R
$d^* = uI + PT - PR$ (Prop. 5.1)	1	$F_S(uI + uR + PT)$
$PR^* = uI + PT - d$, if $uI < d < uI + PT$ (Prop. 5.2)	1	$F_S(uI + uR + PT)$
$\left\{(d, m) \in \Re^2_+ \mid d \leq uI + PT - PR \text{ and } m = uR + PR + d\right\}$ (Prop. 5.3)	$F_S(uI + uR + PT)$	1
$\left\{(d, m) \in \Re^2_+ \mid d = uI + PT - PR \text{ and } m > uR + PR + d\right\}$ (Prop. 5.3)	$F_S(m)$, $m > uI + uR + PT$	$F_S(uI + uR + PT)$
$PR^* = uI + PT - d$, if $((uI<d<uI+PT)\cap(m\geq uI+uR+PT))$ (Prop. 5.4)	$F_S(m)$, $m > uI + uR + PT$	$F_S(uI + uR + PT)$
$PR^* = m - d - uR$, if $((m<uI+uR+PT)\cap(PT+uR>m-d>uR))$ (Prop. 5.4)	$F_S(uI + uR + PT)$	1

Example 3. *Using the data for the total cost in Example 1, assume first that a stop-loss contract with priority d is agreed and that the initial reserves of the insurer and the reinsurer are zero. The premium fixed by the insurer is* 1.8 *(so if the criterion is the expected value, the security loading applied by the insurer is* 80%*). The premium earned by the reinsurer is fixed and equal to* $PR = 0.5, ..., 1.5$*. In Table 5, we calculate the priority that maximizes the joint survival probability, using Proposition 5.1, and the difference between the premium earned by the reinsurer and the expectation of its cost,* $PR - E[SR(d^*)]$*, if the gamma (G), the translated gamma (TG) or the normal approximations (N), are used. In Table 5, we also include the net security premium for the insurer, that is given by* $1.8 - PR - E[SI(d^*)]$*. These two quantities included in Table 5, permit us to calculate the security loading of the reinsurer and the insurer (for the insurer it is the net loading) included in the optimal strategy. These security loadings are shown in Table 6. In Table 7, we calculate the maximal joint survival probability (that equals to the survival probability of the reinsurer (Remark 2)), and the increase in the survival probability of the insurer if the optimal reinsurance is agreed, when the gamma, the translated gamma or the normal approximations, are used.*

Table 5: Priority, security premium for the reinsurer and net security premium for the insurer if the joint survival probability is maximized for several fixed reinsurer's premiums

		$PR - E[SR(d^*)]$			$1.8 - PR - E[SI(d^*)]$		
PR	d^*	G	TG	N	G	TG	N
0.5	1.3	0.1013	0.0820	0.0732	0.6987	0.7180	0.7268
0.6	1.2	0.1750	0.1518	0.1302	0.6250	0.6482	0.6698
0.7	1.1	0.2466	0.2195	0.1844	0.5534	0.5805	0.6156
0.8	1	0.3161	0.2847	0.2358	0.4839	0.5153	0.5642
0.9	0.9	0.3831	0.3474	0.2844	0.4169	0.4526	0.5156
1	0.8	0.4474	0.4072	0.3302	0.3526	0.3928	0.4698
1.1	0.7	0.5087	0.4641	0.3732	0.2913	0.3359	0.4268
1.2	0.6	0.5667	0.5178	0.4134	0.2333	0.2822	0.3866
1.3	0.5	0.6209	0.5679	0.4509	0.1791	0.2321	0.3491
1.4	0.4	0.6706	0.6143	0.4858	0.1294	0.1857	0.3142
1.5	0.3	0.7151	0.6565	0.5181	0.0849	0.1435	0.2819

Table 6: Security loadings of the insurer and the reinsurer if the joint survival probability is maximized for several fixed reinsurer's premiums

		$\frac{100(PR-E[SR(d^*)])}{E[SR(d^*)]}$			$\frac{100(1.8-PR-E[SI(d^*)])}{E[SI(d^*)]}$		
PR	d^*	G	TG	N	G	TG	N
0.5	1.3	25.42	19.61	17.14	116.18	123.38	126.81
0.6	1.2	41.17	33.88	27.71	108.70	117.50	126.34
0.7	1.1	54.40	45.67	35.76	101.24	111.76	127.08
0.8	1	65.31	55.25	41.80	93.78	106.31	129.46
0.9	0.9	74.11	62.85	46.20	86.31	101.18	134.13
1	0.8	80.97	68.70	49.29	78.81	96.44	142.29
1.1	0.7	86.04	72.99	51.34	71.26	92.24	156.26
1.2	0.6	89.49	75.89	52.55	63.61	88.82	181.17
1.3	0.5	91.42	77.57	53.11	55.83	86.63	231.32
1.4	0.4	91.94	78.18	53.14	47.82	86.69	366.30
1.5	0.3	91.12	77.83	52.76	39.44	91.69	1559.80

Table 7: Maximal joint survival probability and the increase in the survival probability of the insurer

	G	TG	N
$\phi^*_{I,R} = \phi_R = F_S(1.8)$	0.8202875	0.7955186	0.7141962
$1 - P[S \leq 1.8]$	0.1797125	0.2044814	0.2858038

As it is reflected in Table 7, obviously, the maximal joint survival probability ($\phi^*_{I,R} = \phi_R = F_S(1.8)$) and the increase in the survival probability of the insurer due to the optimal reinsurance ($1 - P[S \leq 1.8]$), is always the same and is independent of the specific optimal combination of the reinsurer's premium and priority. Hence, from the point of view of the joint survival probability, the reinsurer survival probability and the insurer survival probability, all the alternative combinations of the reinsurer's premium and priority included in Table 6 are indifferent. The differences in the security loading applied by the reinsurer and the net security loading of the insurer do not modify the optimal survival probabilities.

Assume now that the insurer and the reinsurer have positive initial reserves, and that the reinsurer's premium is 0.5 and the total premium is 1.8. From Proposition 5.1, the optimal priority is $d^* = uI + 1.3$, and the maximum joint survival probability is $F_S(uI + uR + 1.8) = \phi^*_{I,R}$. Table 8 includes the optimal priority and the maximum joint survival probability for several combinations of initial capitals, using the translated gamma approximation.

Table 8: d^* and $\phi^*_{I,R}$ as functions of initial capitals, for $PR = 0.5$ and $PT = 1.8$

uI/uR		0.25	0.5	0.75	1
0.25	d^*	1.55	1.55	1.55	1.55
	$\phi^*_{I,R}$	0.855824	0.8788329	0.8981223	0.9143059
0.5	d^*	1.8	1.8	1.8	1.8
	$\phi^*_{I,R}$	0.8788329	0.8981223	0.9143059	0.9278928
0.75	d^*	2.05	2.05	2.05	2.05
	$\phi^*_{I,R}$	0.8981223	0.9143059	0.9278928	0.9393062
1	d^*	2.3	2.3	2.3	2.3
	$\phi^*_{I,R}$	0.9143059	0.9278928	0.9393062	0.9488984

Table 8 shows that when different combinations of initial capitals are considered for a specific uI, the optimal priority does not vary if uR is increased. This result is due to the fact that d^* does not depend on the initial capital of the reinsurer. However, the joint survival probability does change with increasing values.

6. Concluding remarks

In the stop-loss reinsurance contract, the cost of the claims of both the insurer and the reinsurer are related. The correlation coefficient is one of the main measures of dependence between random variables. In this paper, explicit expressions of the correlation coefficient between the cost of the insurer and the cost of the reinsurer are obtained as functions of the parameters of the reinsurance contract (the priority and the maximum).

Two optimal problems with the same objective function, the joint survival probability of the insurer and the reinsurer in one period, are solved. The maximum joint survival probability always exists if the reinsurance premium is fixed, and is equal to the probability that the total cost is less than, or equal, to the sum of the total premium and the two initial capitals. This maximum is attained for a unique value of the priority or for a non-convex set of priority and maximum if the reinsurance contract includes a maximum. If we consider that the parameters of the reinsurance contract are fixed, the optimal reinsurance premium and the maximum joint survival probability do not always exist, and in case they exist, the maximum is exactly the same as in the first problem. These findings can be of great help for the insurer and reinsurer in their decision making process.

Acknowledgements

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